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TEMPERATURE FIELD IN A HEAT-SENSITIVE MULTILAYER HALF-SPACE

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A method is proposed for solving a nonlinear axisymmetrical boundary-value heat-conduction problem for a multilayered half-space with heat transfer, heated by internal heat sources.

When determining the temperature fields in structural elements of radioelectronic equipment, in particular, in manufacture of metalloceramic casings (MCC) at high temperatures, it is important to take into account the dependence of the thermophysical characteristics on the temperature. Such MCC technologies as fusing of semiconductor pastes, applied on ceramic substrates, and brazing of terminals to the latter are implemented at 293-1173 K. Temperature gradients that arise in different parts of ceramic radio components cause microcracks, buckling, exfoliation, bloating, and fracture of semiconductors.

Heat conduction equations for heat-sensitive piecewise-homogeneous bodies are obtained in [1, 2]. Following [2], we use relations and equations describing the nonlinear boundary-value axisymmetric heat conduction problem for a multilayered half-space (Fig. 1) with heat transfer, heated by internal heat sources of power q_0, uniformly distributed throughout the volume of a finite cylinder $\pi R^2(z_{j-1} - z_j)$:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\lambda(t,z)\frac{\partial t}{\partial r}\right] + \frac{\partial}{\partial z}\left[\lambda(t,z)\frac{\partial t}{\partial z}\right] = -q_0 S_-(R-r)N(z), \tag{1}$$

$$\lambda(t,z)\frac{\partial t}{\partial z}\Big|_{z=0} = \alpha_z\left(t|_{z=0} - t_{\text{med}}\right),$$

$$\frac{\partial t}{\partial z}\Big|_{z\to\infty} = 0, \quad t|_{r\to\infty} = 0, \quad \frac{\partial t}{\partial r}\Big|_{r\to\infty} = 0, \tag{2}$$

where

$$\lambda(t,z) = \lambda_1(t) + \sum_{i=1}^{n-1}\left[\lambda_{i+1}(t) - \lambda_i(t)\right]S_-(z-z_i)$$

is the thermal conductivity of the multilayered half-space; $N(z) = S_-(z - z_{j-1}) - S_-(z - z_j)$, $1 \le j < n$.

Using the new function

$$\vartheta = \int_0^{t(r,z)}\lambda_1(\zeta)\,d\zeta + \sum_{i=1}^{n-1}S_-(z-z_i)\int_{t(r,z_i)}^{t(r,z)}\left[\lambda_{i+1}(\zeta) - \lambda_i(\zeta)\right]d\zeta \tag{3}$$

we transform nonlinear problem (1), (2) to the form

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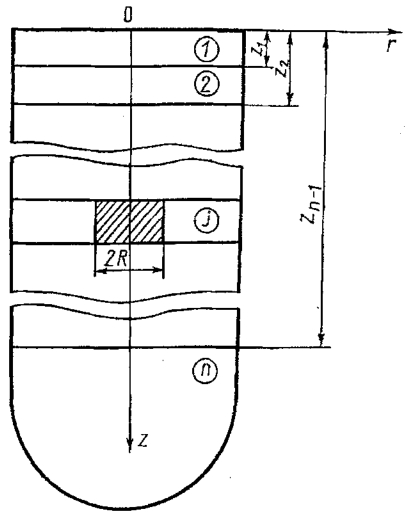

Fig. 1. Multilayer isotropic half-space consisting of n diverse layers, differing in thermophysical and geometric parameters, and referred to the cylindrical coordinate system (r, φ, z) with the origin on the boundary surface.

$$\Delta\vartheta = -\frac{1}{r}\frac{\partial}{\partial r}\left\{ r\sum_{i=1}^{n-1} S_-(z-z_i)\left[(\lambda_{i+1}(t)-\lambda_i(t))\frac{\partial t}{\partial r}\right]\Bigg|_{z=z_i}\right\} - q_0 S_-(R-r)N(z), \tag{4}$$

$$\frac{\partial\vartheta}{\partial z}\Big|_{z=0} = \alpha_z(t|_{z=0}-t_{\text{med}}), \quad \frac{\partial\vartheta}{\partial z}\Big|_{z\to\infty}=0, \quad \vartheta|_{r\to\infty}=0, \quad \frac{\partial\vartheta}{\partial r}\Big|_{r\to\infty}=0. \tag{5}$$

Considering preliminarily the solution of the corresponding linear boundary-value heat conduction problem

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\lambda(z)\frac{\partial t}{\partial r}\right] + \frac{\partial}{\partial z}\left[\lambda(z)\frac{\partial t}{\partial z}\right] = -q_0 S_-(R-r)N(z), \tag{6}$$

$$\lambda_1\frac{\partial t}{\partial z}\Big|_{z=0} = \alpha_z(t|_{z=0}-t_{\text{med}}), \quad t|_{r,z\to\infty}=t_{\text{med}}, \quad \frac{\partial t}{\partial r}\Big|_{r\to\infty}=0, \tag{7}$$

where

$$\lambda(z) = \lambda_1 + \sum_{i=1}^{n-1}(\lambda_{i+1}-\lambda_i)S_-(z-z_i).$$

we will find the character of change of the functions $t(r, 0)$, $t(r, z_i)$.

The linear heat conduction equation for a multilayered half-space with internal heat sources is obtained in [3] in a more general case. Following [3], we may write for our case

$$\Delta T = \sum_{i=1}^{n-1}(\lambda_{i+1}-\lambda_i)\Theta|_{z=z_i}\delta'_-(z-z_i) - q_0 S_-(R-r)N(z). \tag{8}$$

where $T=\lambda(z)\Theta$; $\Theta = t - t_{\text{med}}$.

Applying the Hankel transformation of the coordinate r to Eq. (8) and boundary conditions (7), we arrive at the ordinary differential equation with constant coefficients

$$\frac{d^2\overline{T}}{dz^2} - \xi^2\overline{T} = \sum_{i=1}^{n-1}(\lambda_{i+1}-\lambda_i)\overline{\Theta}|_{z=z_i}\delta'_-(z-z_i) - \frac{q_0 R}{\xi}J_1(R\xi)N(z) \tag{9}$$

and the boundary conditions

$$\lambda_1 \frac{d\overline{T}}{dz}\Big|_{z=0} = \alpha_z \overline{T}\big|_{z=0}, \quad \overline{T}\big|_{z\to\infty} = 0, \tag{10}$$

where

$$\overline{T} = \int_0^{\infty} rJ_0(r\xi)\, T\, dr .$$

Solving Eq. (9) yields the expression

$$\overline{T} = C_1 \exp(\xi z) + C_2 \exp(-\xi z) +$$

$$+ \sum_{i=1}^{n-1} (\lambda_{i+1} - \lambda_i)\, \overline{\Theta}\big|_{z=z_i} \operatorname{ch} \xi (z - z_i)\, S_-(z - z_i) - \frac{q_0 R}{\xi^3} J_1(R\xi)\, f(\xi, z) . \tag{11}$$

Here C_1 and C_2 are integration constants; $f(\xi, z) = \operatorname{ch} \xi(z - z_{j-1})\, S_-(z - z_{j-1}) - \operatorname{ch} \xi(z - z_j)\, S_-(z - z_j) - N(z)$.

The quantities $\overline{\Theta}\big|_{z=z_i}$ are found from the system of linear algebraic equations

$$\overline{T}\big|_{z=z_m} = C_1 \exp(\xi z_m) + C_2 \exp(-\xi z_m) - \frac{q_0 R}{\xi^3} J_1(R\xi) \times$$

$$\times f(\xi, z_m) + \sum_{i=1}^{n-1} (\lambda_{i+1} - \lambda_i)\, \overline{\Theta}\big|_{z=z_i} \operatorname{ch} \xi (z_m - z_i)\, S_-(z_m - z_i)$$

$$(m = 1, 2, \ldots, n-1)$$

in the form

$$\overline{\Theta}\big|_{z=z_i} = C_1 F_i^+(\xi) + C_2 F_i^-(\xi) - \frac{q_0 R}{\xi^3} J_1(R\xi)\, \varphi_i(\xi) , \tag{12}$$

where

$$F_1^{\pm}(\xi) = \exp(\pm \xi z_1)/\lambda_1 ; \quad F_l^{\pm}(\xi) = [\exp(\pm \xi z_l) +$$

$$+ \sum_{i=1}^{l-1} (\lambda_{i+1} - \lambda_i) \operatorname{ch} \xi (z_l - z_i)\, F_i^{\pm}(\xi)]/\lambda_l , \quad l = 2, 3, \ldots, n-1 ;$$

$$\varphi_i(\xi) \equiv 0 , \quad i = 1, 2, \ldots, j-1 ; \quad \varphi_j(\xi) = (\operatorname{ch} \xi (z_j - z_{j-1}) - 1)/\lambda_j ;$$

$$\varphi_l(\xi) = [\operatorname{ch} \xi (z_l - z_{j-1}) - \operatorname{ch} \xi (z_l - z_j) + \sum_{k=j}^{l-1} (\lambda_{k+1} -$$

$$- \lambda_k) \operatorname{ch} \xi (z_l - z_k)\, \varphi_k(\xi)]/\lambda_l , \quad l = j+1, j+2, \ldots, n-1 .$$

Thus, recurrence relations are found for determination of $F_i^{\pm}$, $\varphi_i(\xi)$, which are necessary for determination of $\overline{\Theta}\big|_{z=z_i}$.

From boundary conditions (10) we also derive expressions for the integration constants:

$$C_1 = \frac{\Delta^+}{\Delta} ; \quad C_2 = \frac{\Delta^-}{\Delta} , \tag{13}$$

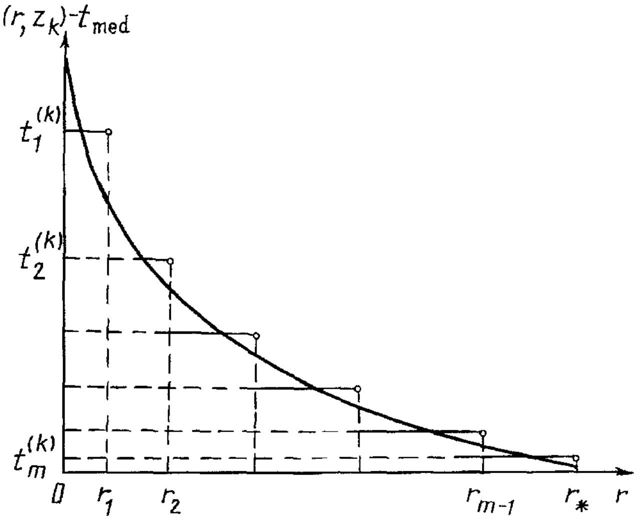

Fig. 2. Approximation of the function $t(r, z_k) - t_{med}$.

where

$$\Delta = \alpha^-(\xi) \sum_{i=1}^{n-1} (\lambda_{i+1} - \lambda_i) F_i^-(\xi) \exp(-\xi z_i) + \alpha^+(\xi) \times$$

$$\times \left(2 + \sum_{i=1}^{n-1} (\lambda_{i+1} - \lambda_i) F_i^+(\xi) \exp(-\xi z_i)\right); \quad \Delta^\pm = \frac{q_0 R}{\xi^3} \alpha^\pm(\xi) P(\xi);$$

$$P(\xi) = \exp(-\xi z_{j-1}) - \exp(-\xi z_j) + \sum_{i=j}^{n-1} (\lambda_{i+1} - \lambda_i) \varphi_i(\xi) \exp(-\xi z_i);$$

$$\alpha^\pm(\xi) = \lambda_1 \xi \pm \alpha_z.$$

Passing to the inverse transform in (11) with account for (12), (13), we may write

$$T = q_0 R \int_0^\infty J_1(R\xi) J_0(r\xi) \Phi(\xi, z) \frac{d\xi}{\xi^2}, \tag{14}$$

where

$$\Phi(\xi, z) = \frac{P(\xi)}{\Delta} [\alpha^+(\xi) \psi^+(\xi, z) + \alpha^-(\xi) \psi^-(\xi, z)] -$$

$$- \sum_{i=j}^{n-1} (\lambda_{j+1} - \lambda_i) \varphi_i(\xi) \operatorname{ch} \xi (z - z_i) S_-(z - z_i) - f(\xi, z);$$

$$\psi^\pm(\xi, z) = \exp(\pm \xi z) + \sum_{i=1}^{n-1} (\lambda_{i+1} - \lambda_i) F_i^\pm(\xi) \operatorname{ch} \xi (z - z_i) S_-(z - z_i).$$

We now represent the functions $t(r, 0)$, $t(r, z_i)$, whose variation is determined by (14), as follows (Fig. 2):

$$t(r, z_k) = \left[t_1^{(k)} + \sum_{l=1}^{m-1} (t_{l+1}^{(k)} - t_l^{(k)}) S_-(r - r_l) \right] S_-(r_* - r). \tag{15}$$

Here $k = 0, 1, 2, \ldots, n-1$; $z_0 = 0$; $r_l \in]0; r_*[$; m is the number of subdivisions of the interval $]0; r_*[$; r_* is the value of the radial coordinate at which the temperature becomes practically constant and equal to t_{med}; $t_l^{(k)}$ are the still unknown temperatures.

Substituting expression (15) into Eq. (4) and boundary condition (5) at the surface $z = 0$, we obtain

$$\Delta\vartheta = -\frac{1}{r}\sum_{i=1}^{n-1}\left[\sum_{l=1}^{m-1} r_l\,(t_{l+1}^{(i)} - t_l^{(i)})\,(\lambda_{i+1}\,(t_{l+1}^{(i)}) - \lambda_i\,(t_{l+1}^{(i)}))\,\times\right.$$
$$\left.\times\,\delta'_-\,(r - r_l) + r_*\,t_m^{(i)}\,(\lambda_{i+1}\,(t_m^{(i)}) - \lambda_i\,(t_m^{(i)}))\,\delta'_-\,(r_* - r)\right] S_-\,(z -$$
$$- z_i) - q_0 S_-\,(R - r)\,N(z)\,, \tag{16}$$

$$\left.\frac{\partial\vartheta}{\partial z}\right|_{z=0} = \alpha_z\left\{\left[t_1^{(0)} + \sum_{l=1}^{m-1}(t_{l+1}^{(0)} - t_l^{(0)})\,S_-\,(r - r_l)\right] S_-\,(r_* - r) - t_{med}\right\},$$
$$\left.\frac{\partial\vartheta}{\partial z}\right|_{z\to\infty} = 0\,,\quad \vartheta\big|_{r\to\infty} = 0\,,\quad \left.\frac{\partial\vartheta}{\partial r}\right|_{r\to\infty} = 0\,. \tag{17}$$

Having applied the integral Hankel transformation of the coordinate r to boundary-value problem (16), (17), we arrive at the ordinary differential equation with constant coefficients

$$\frac{d^2\bar\vartheta}{dz^2} - \xi^2\bar\vartheta = -\xi\sum_{i=1}^{n-1}\left[\sum_{l=1}^{m-1} r_l J_1\,(r_l\xi)\,(t_{l+1}^{(i)} - t_l^{(i)})\,(\lambda_{i+1}\,(t_{l+1}^{(i)}) -\right.$$
$$\left.- \lambda_i\,(t_{l+1}^{(i)})) + r_* J_1\,(r_*\xi)\,t_m^{(i)}\,(\lambda_{i+1}\,(t_m^{(i)}) - \lambda_i\,(t_m^{(i)}))\right] \times S_-\,(z - z_i) - \frac{q_0 R}{\xi} J_1\,(R\xi)\,N(z) \tag{18}$$

and the boundary conditions

$$\left.\frac{d\bar\vartheta}{dz}\right|_{z=0} = \frac{\alpha_z}{\xi}\left[r_* J_1\,(r_*\xi)\,t_m^{(0)} - t_{med}\delta_+\,(\xi) - \sum_{l=1}^{m-1} r_l\,\times\right.$$
$$\left.\times\,J_1\,(r_l\xi)\,(t_{l+1}^{(0)} - t_l^{(0)})\right],\quad \left.\frac{d\bar\vartheta}{dz}\right|_{z\to\infty} = 0\,, \tag{19}$$

where

$$\bar\vartheta = \int_0^\infty rJ_0\,(r\xi)\,\vartheta dr\,.$$

Solving boundary-value problem (18), (19) and then passing to the inverse transform using the transformation formula, we obtain the following expression for the function ϑ:

$$\vartheta = \frac{1}{2}\left\{\sum_{i=1}^{n-1}\left[\sum_{l=1}^{m-1} r_l\,(t_{l+1}^{(i)} - t_l^{(i)})\,(\lambda_{i+1}\,(t_{l+1}^{(i)}) - \lambda_i\,(t_{l+1}^{(i)}))\int_0^\infty G\,(r_l\,,\,r\,,\,\xi)\,\times\right.\right.$$
$$\left.\times\,H\,(\xi\,,\,z)\,d\xi + r_* t_m^{(i)}\,(\lambda_{i+1}\,(t_m^{(i)}) - \lambda_i\,(t_m^{(i)}))\int_0^\infty G\,(r_*\,,\,r\,,\,\xi)\,H\,(\xi\,,\,z)\,d\xi\right] +$$

$$+ q_0 R \int_0^\infty G(R, r, \xi) [2N(z) - \exp(-\xi |z - z_{j-1}|_-) \operatorname{sign}_-(z - z_{j-1}) +$$

$$+ \exp(-\xi |z - z_j|_-) \operatorname{sign}_-(z - z_j) + \exp(-\xi (z + z_{j-1})) -$$

$$\left. - \exp(-\xi (z + z_j))] \frac{d\xi}{\xi^2} \right\} - \alpha_z \left[r_* t_m^{(0)} \int_0^\infty G(r_*, r, \xi) \exp(-\xi z) \frac{d\xi}{\xi} - \right.$$

$$\left. - \sum_{k=1}^{m-1} r_k (t_{k+1}^{(0)} - t_k^{(0)}) \int_0^\infty G(r_*, r, \xi) \exp(-\xi z) \frac{d\xi}{\xi} + t_{\text{med}} z \right], \tag{20}$$

where

$$H(\xi, z) = \exp(-\xi (z + z_i)) - \exp(-\xi |z - z_i|_-) \operatorname{sign}_-(z - z_i) +$$

$$+ 2S_-(z - z_i); \quad G(\alpha, r, \xi) = J_0(r\xi) J_1(\alpha\xi).$$

Introducing concrete dependences of the thermal conductivities of conjugated elements into (3), (20) and comparing the obtained expressions for the function ϑ on the surfaces $z = 0, z_1, z_2, \ldots, z_{n-1}$ yields a system of nonlinear algebraic equations for determining the unknown temperatures $t_l^{(k)}$.

The sought temperature field for the system under consideration is determined from nonlinear algebraic equations obtained from (3) and (20) after substituting in them expressions for the dependences of the thermal conductivities of the layers of the half-space.

As an example, we consider a three-layer half-space with heat sources in the second layer. In this case, $n = 3$, $j = 2$. In many practical cases [4] the thermal conductivity depends linearly on the temperature, $\lambda = \lambda^0(1 - kt)$, where λ^0 and k are the reference and temperature coefficients of thermal conductivity.

The solution of the corresponding linear problem for our example is written in the form of (14), where the function $\Phi(\xi, z)$ is:

in the domain $0 \le z < z_1$

$$\Phi(\xi, z) = \frac{1}{\Delta_1} \left\{ \alpha^+(\xi) \left[\frac{1}{2} (K_2^- \exp(\xi (z + z_1 - 2z_2)) + \right. \right.$$

$$\left. + K_2^+ \exp(\xi (z - z_1))) - K_\lambda^{(2)} \exp(\xi (z - z_2)) \right] + \tag{21}$$

$$+ \alpha^-(\xi) \left[\frac{1}{2} (K_2^- \exp(\xi (z_1 - z - 2z_2)) + K_2^+ \exp(-\xi (z + \right.$$

$$\left. \left. + z_1))) - K_\lambda^{(2)} \exp(-\xi (z + z_2)) \right] \right\},$$

in the domain $z_1 \le z < z_2$

$$\Phi(\xi, z) = \frac{1}{2\Delta_1} \left\{ \alpha^+(\xi) [K_1^- (K_2^- \exp(2\xi (z_1 - z_2)) - K_\lambda^{(2)} \times \right.$$

$$\times \exp(\xi (2z_1 - z_2 - z))) + K_2^- \exp(\xi (z_1 - 2z_2 - z)) -$$

$$- K_1^+ K_\lambda^{(2)} \exp(\xi (z - z_2)) + K_2^+ (K_1^- - \exp(\xi (z_1 - z)))] +$$

$$+ \alpha^-(\xi) [K_1^- (K_2^+ \exp(-2\xi z_1) - K_\lambda^{(2)} \exp(\xi (z - 2z_1 - z_2))) +$$

$$+ K_1^+ (K_2^- \exp(-2\xi z_2) - K_\lambda^{(2)} \exp(-\xi (z + z_2))) -$$

$$- K_2^- \exp(\xi (z - z_1 - 2z_2)) + K_2^+ \exp(-\xi (z + z_1))]\}, \qquad (22)$$

in the domain $z_2 \le z < \infty$

$$\Phi(\xi, z) = \frac{K_\lambda^{(2)}}{\Delta_1} \left\{ \alpha^+(\xi) \left[\left(1 + \frac{1}{2} K_1^-\right) \exp(\xi (z_2 - z)) - \right.\right.$$

$$\left. - \exp(\xi (z_1 - z)) - \frac{1}{2} K_1^- \exp(\xi (2z_1 - z_2 - z)) \right] +$$

$$+ \alpha^-(\xi) \left[\exp(-\xi (z + z_1)) + \frac{1}{2} K_1^- \exp(\xi (z_2 - 2z_1 - z)) - \right.$$

$$\left.\left. - \left(1 + \frac{1}{2} K_1^-\right) \exp(-\xi (z + z_2)) \right]\right\}. \qquad (23)$$

Here

$$\Delta_1 = \frac{1}{2} [\alpha^-(\xi) (K_1^- K_2^+ \exp(-2\xi z_1) + K_1^+ K_2^- \exp(-2\xi z_2)) +$$

$$+ \alpha^+(\xi) (K_1^+ K_2^+ + K_1^- K_2^- \exp(2\xi (z_1 - z_2)))]; \quad K_l^\pm = K_\lambda^{(l)} \pm 1, \quad l = 1, 2. \qquad (24)$$

Next, using [4] we will have for the dimensionless temperature $T_* = t/t_{med}$:
for the domain $0 \le z < z_1$

$$T_* = (1 - \sqrt{1 - 2K_1 \, \mathrm{Ki} \, \Theta^* K_{\lambda^\circ}^{(1)}}) / K_1, \qquad (25)$$

for the domain $z_1 \le z < z_2$

$$T_* = (1 - \sqrt{1 - 2K_2 (\mathrm{Ki} \, \Theta^* + \Theta_1^*)}) / K_2, \qquad (26)$$

for the domain $z_2 \le z < \infty$

$$T_* = (1 - \sqrt{1 - 2K_3 (\mathrm{Ki} \, \Theta^* + \Theta_2^*) / K_{\lambda^\circ}^{(2)}}) / K_3, \qquad (27)$$

where

$$\Theta^* = \vartheta / (q_0 R^2); \quad \Theta_1^* = \left[T_1 \left(1 - \frac{1}{2} K_2 T_1 - \frac{1}{K_{\lambda^\circ}^{(1)}} \left(1 - \frac{1}{2} K_1 T_1\right)\right)\right]\Bigg|_{z=z_1};$$

$$T_1 |_{z=z_l} = (1 - \sqrt{1 - 2K_1 \, \mathrm{Ki} \, \Theta^* |_{z=z_l} K_{\lambda^\circ}^{(1)}}) / K_1;$$

$$\Theta_2^* = \left[T_2 \left(K_{\lambda^\circ}^{(2)} \left(1 - \frac{1}{2} K_3 T_2\right) - 1 + \frac{1}{2} K_2 T_2 \right)\right]\Bigg|_{z=z_2} +$$

$$+ \left[T_2 \left(1 - \frac{1}{2} K_2 T_2 - \frac{1}{K_{\lambda^\circ}^{(2)}} \left(1 - \frac{1}{2} K_1 T_2\right)\right)\right]\Bigg|_{z=z_1};$$

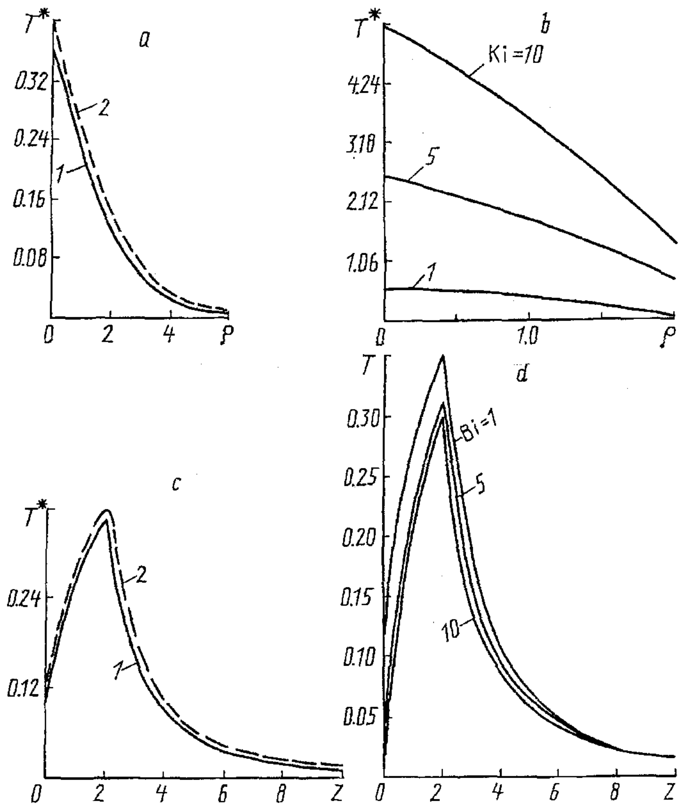

Fig. 3. Dimensionless temperature T^* as a function of the dimensionless coordinate ρ at Bi = 1, Ki = 1, Z = 1 (a) and at Bi = 1, Z = 2 (b) and the dimensionless coordinate Z at Bi = 1, Ki = 1, ρ = 1 (c) and at Ki = 1, ρ = 1 (d).

$$T_2|_{z=z_2} = \left(1 - \sqrt{1 - 2K_2\left(\mathrm{Ki}\,\Theta^*|_{z=z_2} + \Theta_3^*\right)}\right)/K_2\,;$$

$$\Theta_3^* = \left[T_2\left(1 - \frac{1}{2}K_2T_2 - \frac{1}{K_{\lambda^\circ}^{(1)}}\left(1 - \frac{1}{2}K_1T_2\right)\right)\right]\Bigg|_{z=z_1}\,;$$

$$K_i = k_i t_{\mathrm{med}}\,,\quad i = 1\,,\,2\,,\,3\,,\;;\quad K_{\lambda^\circ}^{(l)} = \lambda_{l+1}^0/\lambda_l^0\,,\quad l = 1\,,\,2\,; \qquad (28)$$

$T_2|_{z=z_1}$ is determined from an algebraic equation by substituting (28) into the expression

$$\left\{T_2\left[1 - \frac{1}{2}K_2T_2 - K_{\lambda^\circ}^{(2)}\left(1 - \frac{1}{2}K_3T_3\right)\right]\right\}\Bigg|_{z=z_2} +$$

$$+ \left\{T_2\left(K_{\lambda^\circ}^{(2)}\left(1 - \frac{1}{2}K_3T_2\right) - 1 + \frac{1}{2}K_2T_2 \right. +\right.$$

$$\left.\left. + \frac{1}{K_{\lambda^\circ}^{(1)}}\left(1 - \frac{1}{2}K_1T_2\right)\right]\right\}\Bigg|_{z=z_1} = \mathrm{Ki}\,\Theta^*|_{z=z_l}\,.$$

Using formulas (25)-(27) at $m = 5$ in the interval 300-1500 K and formula (14) with account for expressions (21)-(24), we performed numerical calculations and investigated distributions of the dimensionless temperature $T^* = t/t_{\mathrm{med}} - 1$ for the following materials: tungsten, the 1st layer; molybdenum, the 2nd layer; ceramics VK94-1, the 3rd layer [5].

Figure 3 shows the dimensionless temperature T^* as a function of the radial $\rho = r/R$ and axial $Z = z/R$ coordinates. As is seen from the results in Fig. 3a, c, account for the dependence of the thermal conductivities on the temperature (curve 1) leads to its decrease as compared to a nonheat-sentitive system (the thermophysical parameters are independent of the temperature, curve 2) by 3% for the above materials.

The temperature distributions over the coordinates ρ and z at different Ki and Bi values in Fig. 3b, d show that with increasing Kirpichev number the temperature increases substantially in the region where the heat sources act, and at the dimensionless coordinate $z \geq 8$ it is practically independent of the heat transfer coefficient.

NOTATION

$t(r, z)$, temperateure field; λ_i, thermal conductivity of the i-th layer of the half-space; $z = z_i$, conjugation surface of the i-th and $(i + 1)$-th layers; α_z, heat transfer coefficient from the surface $z = 0$; t_{med}, temperature of the outer medium; $S_\pm(\zeta)$, asymmetric unit functions; $J_\nu(\zeta)$, ν-th order Bessel function of the first kind; $\text{Ki} = q_0R^2/(\lambda_0^2 t_{\text{med}})$, Kirpichev number; $\text{Bi} = \alpha_z z_1/\lambda_1^0$, Biot number; $K_\lambda^{(I)}$, criterion characterizing the relative heat conductivity of the layers of the half-space; $\Delta = (1/r)(\partial/\partial r)(r\partial/\partial r) + \partial^2/\partial z^2$, Laplace operator; $\delta_\pm(\zeta) = dS_\pm(\zeta)/d\zeta$, $|\zeta|_- = \begin{cases} \zeta, & \zeta \geq 0, \\ -\zeta, & \zeta < 0; \end{cases}$ $\text{sign}_-(\zeta) = \begin{cases} 1, & \zeta \geq 0, \\ -1, & \zeta < 0. \end{cases}$

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